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Continuous Stern–Gerlach effect: Principle and idealized apparatus

(trapped single electron/axial oscillation frequency/spin-dependent classical trajectories/reduction of wavefunction/measurement time)

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ABSTRACT A nondestructive type of Stern–Gerlach effect for an individual electron is described that has been thoroughly demonstrated in experiments at the University of Washington. This "continuous Stern–Gerlach effect" makes use of an inhomogeneous magnetic field provided by a weak auxiliary magnetic bottle and is nondestructive in contrast to all previous versions of the effect. As in the classic Stern–Gerlach effect, changes in the spin state are detected via changes in classical particle trajectories; however, what is observed now is not a deflection of the orbit but rather a change of its frequency in the storage well. A simplified model of the continuous Stern–Gerlach effect at zero temperature is introduced to develop the relation between minimum measurement time required for determination of spin state, driven oscillation amplitude, and zero-point fluctuations in the storage well. The problem of the reduction of the wavefunction by the interaction of the electron with the apparatus is addressed following W. Pauli.

Van Dyck, Schwinberg, and I (1, 2) have described a series of experiments, in which a nondestructive type of Stern–Gerlach effect has been thoroughly demonstrated and put to good use. The present paper now attempts a timely, fuller discussion of this nondestructive "continuous Stern–Gerlach effect" (CSGE) than has been offered in the past (3). Also it is believed that the CSGE experiments, which are amenable to quantitative, step-by-step analysis, provide a particularly clean illustration of the idealized measurement process, which constitutes the basis of quantum mechanics. This is not particularly surprising as the classic (transient) Stern–Gerlach effect is one of its historical cornerstones. For a brief historical sketch of earlier attempts to adapt the Stern–Gerlach effect to electrons, the reader is referred to ref. 3. The second section of this paper, *Principle*, spells out the principles underlying the continuous effect by comparing it to the classic effect. In the section *Idealized ω_z-Shift Spectrometer* a somewhat idealized measuring apparatus is analyzed focusing not on "observables" but on actually observed quantities. The important limitations of the measurement process by thermal and zero-point fluctuations will be discussed in a second paper (4).

Principle

The famous Stern–Gerlach experiment (5) on a beam of silver atoms passing through a transverse inhomogeneous magnetic field had shown that, when a parameter of the trajectory (the deflection) becomes known accurately enough—from silver deposits on a glass plate—to deduce from it the z component of the magnetic moment, the values found are only $\pm\mu_B$. In our CSGE experiment on an individual electron we greatly increased the detection sensitivity for changes in the classical trajectory, which is now essentially parallel to the magnetic field. This has been achieved by electronic techniques and by making the trajectory periodic in an axial electric parabolic trapping potential and very long (see Fig. 1). A weak, auxiliary magnetic bottle—known from Lawrence's cyclotron—produced by a nickel wire ring magnetized to saturation causes the axial oscillation frequency, $\omega_z \simeq 2\pi \times 60$ MHz, to be slightly spin dependent

$$\omega_z(\uparrow) - \omega_z(\downarrow) \equiv \delta \approx 2\pi \times 1 \text{ Hz}. \quad [1]$$

See Fig. 2. The parameters of the bottle field are

$$b_z = \beta(z^2 - r^2/2),\ b_y = -\beta zy,\ b_x = -\beta zx, \quad [2]$$

$$D_z^m = \beta Z_0^2\, \mu_B \approx 0.12\ \mu\text{eV},\quad \beta \approx 120 \text{ G/cm}^2, \quad [3]$$

with D_z^m the depth (for magnetic moment down) of the bottle-well between the cap electrodes separated by $2Z_0$. For the corresponding frequency shift δ for a spin flip we may write,

$$\delta \approx (D_z^m/D_z^e)\omega_z, \quad [4]$$

which, for the electric well depth $D_z^e \approx 5$ eV, gives $\delta \approx 1.4$ Hz. The determination of the spin direction then takes the form of a resonance frequency measurement. It appears that our technique is an example par excellence of the ideal quantum-mechanical measuring process.

(*i*) It is carried out on an essentially free individual particle (electron) whose spin relaxation time is practically infinite.

(*ii*) The measurement may be repeated on the same particle as often as one likes or even continuously.

(*iii*) Questions of measurement theory, such as Zeno's paradox, may be studied experimentally.

The quantum-mechanical measurement problem in general and in the classic, transient Stern–Gerlach effect (TSGE) in particular has been addressed extensively in the literature (6–9). From Pauli's discussion the following points may be gleaned for our purpose.

(*a*) In the TSGE the motion of the center of mass of the atom (in combination with a position detector) may be used as a measurement device for the spin.

(*b*) By *postulate* an appropriate nondestructive position measurement in the TSGE reduces the general spin wavefunction

$$\psi = \alpha|\uparrow\rangle + \beta|\downarrow\rangle,\quad \alpha\alpha^* + \beta\beta^* = 1 \quad [5]$$

to

$$\psi = |\uparrow\rangle \text{ or } \psi = |\downarrow\rangle, \quad [6]$$

with respective probabilities $\alpha\alpha^*$, $\beta\beta^*$, where $|\uparrow\rangle$, $|\downarrow\rangle$ are spin eigenfunctions for $m = \pm 1/2$. Such a nondestructive position measurement has not been realized in the laboratory so far.

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Abbreviations: CSGE, continuous Stern–Gerlach effect; TSGE, transient Stern–Gerlach effect; PSD, phase-sensitive detector.

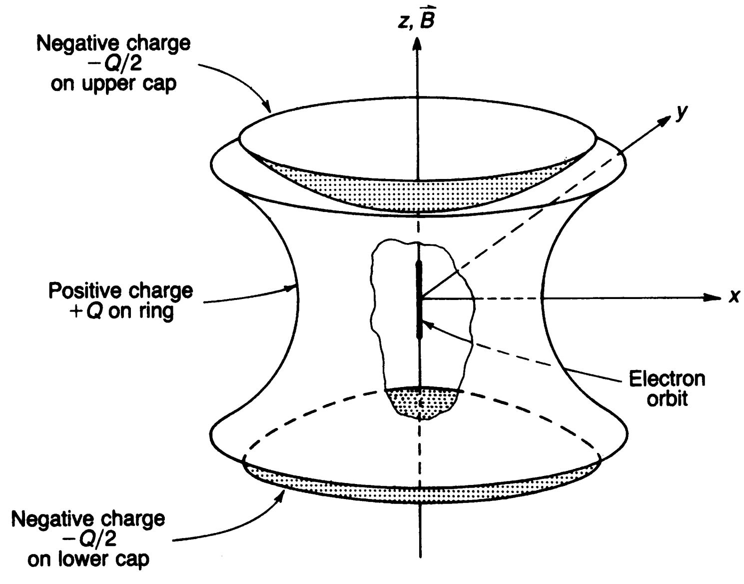

FIG. 1. Electron in Penning trap, the Geonium atom. In the simple mono-electron oscillator mode shown, the electron moves only parallel to the magnetic field B and along the symmetry axis of the electrode structure. Each time it gets too close to one of the negatively charged cap electrodes, it is turned around and a rf oscillatory motion results.

(*c*) If a (nondestructive) measurement on a stationary state is repeated, it gives the same result as the preceding one.

(*d*) To complete a measurement a time, T_m, is needed.

(*e*) The measurement may be carried out by an automated recording mechanism.

(*f*) In general, for the measurement of the center of mass coordinate any other suitable experimental scheme that clearly correlates spin direction with the indication of a meter may be substituted.

(*g*) If the result of the measurement is not announced, state reduction to either $|\uparrow\rangle$ or $|\downarrow\rangle$ nevertheless does take place.

From the above follows:

(*h*) For the completion of the measurement process no animate observer is needed.

(*i*) Our CSGE scheme does determine the spin direction and reduces the wavefunction.

(*j*) Since completion of an optimal measurement takes a minimum time $T_m{}^*$, state reduction cannot occur sooner. However, since the measuring apparatus may be less than optimal, the time for state reduction T_r may be shorter than T_m for the nonoptimal measurement.

(*k*) To effect state reduction without announcement of the result of the measurement, the interaction of the electron with very few essential components of the measuring apparatus is sufficient.

(*l*) Transition processes occurring in a time $<< T_r$ are very little affected by the measuring process. A lower limit for T_r may be obtained from the linewidth observed in our experiments when using *continuous* spin-state measurement. This resolves Żeno's paradox in quantum mechanics (10), which rests on the unjustified assumption of *instantaneous* measurement, whereas Pauli has demonstrated the need of a finite time for measurement and consequently state reduction.

Idealized ω_z-Shift Spectrometer

To illustrate our technique, consider an idealized apparatus, which, however, contains all of the essential elements of such a spectrometer. These elements (see Fig. 3) are

(*i*) an undamped series resonant ℓc circuit representing the elastically bound electron oscillating at ω_z between the caps of a very small Penning trap of negligible capacity (11);

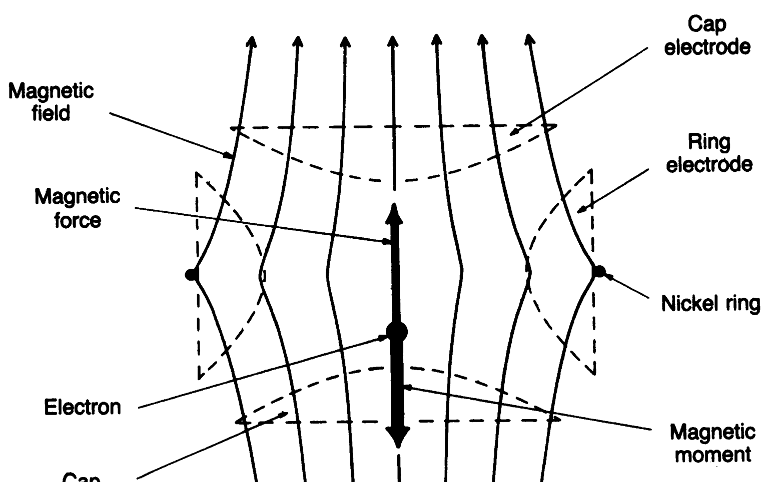

FIG. 2. CSGE (schematic). The electron moving along a field line in a large, ≈5 tesla, magnetic field of a small, shaped inhomogeneity, with its spin up and its magnetic moment antiparallel to the field, is driven toward weaker fields. Shown here is the minute magnetic force, which adds to the much stronger electric restoring force and slightly deepens the net axial trapping potential. As a consequence, the axial oscillation frequency, which has the value $\omega_z \approx 2\pi \times 60$ MHz in the absence of the magnetic bottle inhomogeneity, is increased to $\omega_z(\uparrow) = \omega_z + ½\delta$, $\delta = 2\pi \times 1.4$ Hz. Analogously, for spin down the axial oscillation frequency is reduced to $\omega_z(\downarrow) = \omega_z - ½\delta$. In the classic Stern–Gerlach effect, for silver atoms of the same velocity deposited on the glass plate, only discrete displacements $+a$ and $-a$ were found; thus, for the trapped electron only the discrete shifts $+½\delta$ and $-½\delta$ of the axial oscillation frequency are observed. Effects of the zero-point cyclotron and magnetron motions have been neglected here.

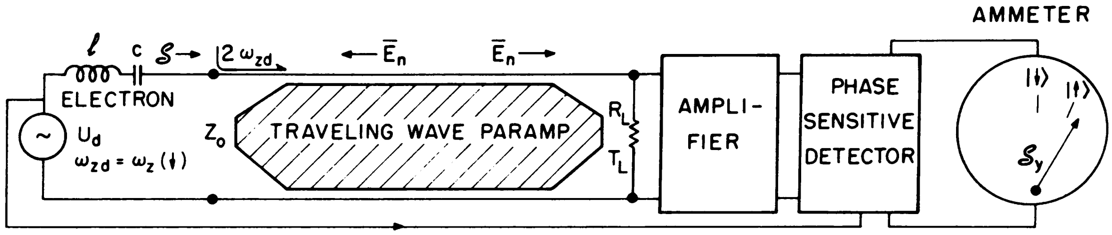

FIG. 3. Equivalent circuit diagram of idealized ω_z-shift spectrometer. The electron, assumed to oscillate, with its spin down, in a tiny Penning trap of negligible capacity, is represented by the $\ell c Z_0$ circuit series-resonant at $\omega_z(\downarrow)$. Here Z_0 denotes the impedance of the very long transmission line, which also acts as parametric amplifier. On resonance, $\omega_{zd} = \omega_z(\downarrow)$, the *rf* source u_d, of fixed frequency ω_{zd}, causes a sharply resonant signal current S to flow into the amplifier. The further amplified signal is fed into a phase-sensitive detector (PSD), whose output is displayed on a milliampere meter. The meter reads $S_y = 0$, because the reference phase is set for detection of the in-quadrature or dispersion signal, which vanishes on resonance. For spin up $\omega_z(\downarrow)$ changes to the slightly off-resonant value $\omega_z(\uparrow)$. In this case a finite in-quadrature signal appears, and the meter reads a finite value S_y corresponding to spin up.

(*ii*) a sinusoidal drive voltage, $u_d = u_{d0}\cos\omega_{zd}t$, provided by a large loss-free resonant cavity storing a very large amount of electromagnetic energy, ω_{zd} fixed at $\omega_z(\downarrow) \approx 2\pi \times 60$ MHz;

(*iii*) a resistive impedance R_L completing the $\ell c R_L$ resonant circuit whose frequency shift δ is to be measured. Here R_L, whose temperature, T_L, may be lowered to approach zero, is the input impedance of a very long loss-free dielectric transmission line of characteristic impedance $Z_0 = R_L$ terminated free of reflections;

(*iv*) noise-free (12) parametric amplification (13, 14) achieved by injecting via directional coupler and copropagating a strong signal at $2\omega_{zd}$ with the ω_{zd} signal on the long transmission line, whose filling medium has a highly electric field-sensitive dielectric constant (chosen for concreteness, any other analyzable zero-noise preamplifier could be substituted here);

(*v*) a zero-noise $2\omega_{zd}$ source similar to *ii*;

(*vi*) a postamplifier operating at room temperature, which adds no significant noise and does not perturb the electron;

(*vii*) a PSD, which may be imagined as a commutating switch synchronous with u_d followed by a low-pass filter, and integrator;

(*viii*) a recording current meter displaying the PSD output and calibrated to read the amplitude of the coherent signal current S through the $\ell c R_L$ circuit.

The drive voltage u_d causes a sharply resonant signal current S to flow through the $\ell c R_L$ circuit. We decompose S into components S_x, S_y, in phase and in quadrature with u_d,

$$S = S_x\cos\omega_{zd}t - S_y\sin\omega_{zd}t, \quad [7]$$

and focus on S_y. This "dispersion" component vanishes on resonance, $\omega_{zd} = \omega_z(\downarrow)$, and is proportional to the small detuning δ to be measured. Width γ_z and decay time τ_z of the electron resonance are given by

$$\gamma_z = R_L/\ell, \ \gamma_z\tau_z = 1. \quad [8]$$

Near resonance we have

$$S \approx u_d/R_L \approx S_x\cos\omega_{zd}t. \quad [9]$$

For a slight shift in the electron frequency to

$$\omega_z(\uparrow) = \omega_{zd} + \delta, \ \delta << \gamma_z, \quad [10]$$

the signal component takes the value

$$S_y \approx 2\delta S_x/\gamma_z. \quad [11]$$

The corresponding phase shift is

$$\phi_\delta \approx S_y/S_x \approx 2\delta/\gamma_z. \quad [12]$$

Thus, the classical electron orbits for the spin states $|\uparrow\rangle$ and $|\downarrow\rangle$ will be forced oscillations of nearly the same amplitude z_0 but with a relative phase shift ϕ_δ and a (periodic) maximum spatial separation

$$\zeta_{\uparrow\downarrow} \approx 2\delta z_0/\gamma_z. \quad [13]$$

In the apparatus the signal S_y launches a wave down the very long transmission line. The amplitude of this wave increases exponentially with propagation distance and the greatly amplified wave is finally completely absorbed by the termination R_L. The input impedance Z_0 of the line is unaffected by the amplification process. The very cold, small termination resistance R_L shunts the extremely large, room temperature input impedance of the postamplifier, whose output is processed by the PSD. The output of the latter, after integration for consecutive periods T_m, is displayed at times $t = T_m, 2T_m, 3T_m \ldots$ on the meter. This meter reads $S_y = S_{y\downarrow} = 0$ for spin down and $S_y = S_{y\uparrow} = 2u_{d0}\delta/R_L\gamma_z$ for spin up (see Fig. 3). Previous experiments (1) show that the in-phase on-resonance or absorption mode signal $S_x = u_{d0}/R_L$ is larger than the background noise by a factor of about 20. Thus, with our choice of $2\delta/\gamma_z \approx 1/2$, the spin-state indicator signal $S_{y\uparrow}$ is expected to be comfortably larger than the noise also.

In conclusion, I have developed the analogy between the classic (transient) Stern–Gerlach effect and the CSGE. I have constructed and analyzed a somewhat idealized model apparatus that detects the spin state of an individual trapped electron by a precise measurement of the extraordinarily sharp axial oscillation frequency of this electron in the confining well. This well incorporates a spin-dependent weak magnetic trapping or antitrapping potential produced by an appropriately shaped inhomogeneous Stern–Gerlach-type field. The apparatus generates a detectable signal current that may be read by a milliampere meter indicating either spin up or spin down.

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